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ESSENCE OF DARK MATTER

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Сущность темной материи

Правая сторона уравнения Эйнштейна является чисто феноменологической. Обращается внимание на то, что в сформулированной Лоренцем теории электронов правая сторона уравнений Максвелла также является чисто феноменологической. Однако с открытием полевой, или квантовой, электродинамики, основанной на уравнениях Максвелла–Шредингера и Максвелла–Дирака, детали правой стороны уравнений Максвелла были прояснены на фундаментальном теоретико-полевом уровне. Сформулирована полевая гравитодинамика, установлены основные понятия и уравнения полевой, или квантовой, гравитодинамики и, следовательно, раскрыта сущность так называемой темной материи.

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Essence of Dark Matter

The right-hand side of the Einstein equation is purely phenomenological. We note that in the Lorentz theory of electrons the right-hand side of the Maxwell equations is purely phenomenological as well. But with the discovery of field, or quantum, electrodynamics, based on the Maxwell–Schrödinger and Maxwell–Dirac equations, the details of the right-hand side of the Maxwell equations were clarified on the fundamental field-theoretical level. In this paper, we formulate a field theory of the gravitodynamics. The main notions and equations of the field, or quantum, gravitodynamics are established and, hence, the essence of the so-called dark matter is disclosed.

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INTRODUCTION

The idea of constructing a theory of gravitation on the basis of field concept was put forward by Einstein. In 1913, the tensor gravitational field was introduced [1], which enabled explaining the rotation of Mercury's perihelion in 1915. Subsequent research by Einstein has been actually aimed at finding a field that could be the carrier of gravitational charge in some way analogous to the electric one. The search for a field gravitodynamics continued till 1955 [2] and was not completed.

The idea of gauge or internal symmetry, first proposed by Weyl in 1918 on the basis of the general covariant concepts of the Einstein theory, gained fundamental importance in the Standard Model but did not find heuristic application in general relativity itself. The internal symmetry transformations are inseparable from field representations since they concern field functions and do not affect coordinates. Thus, internal symmetry is related with phenomena that cannot be described based on the classical concepts, for example, the concept of trajectory and, hence, point particle.

The left-hand side of the Einstein equation is defined by the curvature tensor of the gravitational field and, hence, it is very beautiful from a geometrical point of view. Einstein believed that the right-hand side of his equation should be a perfect expression in gravitodynamics as well. That is why Einstein looked for new fields with the goal to replace the phenomenological right-hand side of his equation as a temporary way out. Our goal here is to formulate the main equations of gravitodynamics and by this to disclose a field-theoretical content of the right-hand side of the Einstein equation. The left-hand side stays without any changes. Thus, we associate the so-called dark matter with the solution of the Einstein problem. Within the field theory of gravity, a theoretical description of dark matter is conducted. Evidence of dark matter was discovered nearly a century ago, but until now only gravitational interaction of this matter was confirmed.

The paper is organized as follows. In the first section, we first of all represent important aspects of the principle of general covariance. After that we consider the parallel displacement defined by the metric and covariant derivative associated with this displacement. The commutator of the covariant derivatives gives the curvature tensor of the gravitational field which defines the left-hand side of the Einstein equation. Following this line of ideas, we consider the most general parallel displacement (quite independent of the metric and anything else) and recognize that it is tightly connected with a group of general covariant gauge symmetry G_c, which defines a general covariant gauge field and has vector and covector fields as fundamental two-valued representation. The commutator of gauge covariant derivatives gives the curvature tensor of the general covariant gauge field and the needed details of the right-hand side of the Einstein equation. The notion of the ground state of the general covariant gauge field is introduced. In the second section, we consider the curvature tensor of the general covariant gauge field and its two-valued representation in more detail. The curvature tensor of the gravitational field is traceless, but the curvature tensor of the general gauge field has a trace. Hence, we separate the gauge invariant trace part and deal with it separately. We derive equations of gravitodynamics and demonstrate that the trace part represents the electromagnetic field.

In the conclusion, we discuss some physical aspects of gravitodynamics.

1. GENERAL COVARIANT GAUGE FIELD

In what follows we will often appeal to the principle of general covariance and that is why we pay attention to three important aspects of this principle. A full correspondence with the principle of general covariance is passed on saying “general covariant”.

In accordance with the seminal work of Einstein and Grossmann [1], this principle, before all, means that functions of multiple variables are elementary objects and hence, to formulate the laws of Nature, it is possible to use only systems of functions (invariants) which are classified by the laws of transformation under the transition from initial system of coordinates to any other one. An important element of the theory of functions of multiple variables and, hence, general covariant theory is the reference space R^n. The point of reference space is defined as an n-tuple of given real numbers $x = (x^1,\, x^2, \cdots x^n)$, $-\infty < x^i < \infty$, and the distance function is introduced as usual:

$$d(x,y) = \sqrt{(x^1-y^1)^2 + (x^2-y^2)^2 + \cdots + (x^n-y^n)^2}\,.$$

One more aspect of the principle of general covariance was discovered by Einstein as general relativity or reparameterization symmetry (R-symmetry) in [3]. To define this symmetry, let us consider a domain Ω in the reference space R^n. By the transformation of this domain, we will call $2n$ real functions $\alpha^i(x^1,\cdots,x^n) = \alpha^i(x)$, $\alpha^i_{-1}(x^1,\cdots,x^n) = \alpha^i_{-1}(x)$ for which the domains of their definition coincide with Ω and moreover they define one-to-one mapping of Ω onto itself so as to $y^i = \alpha^i(x)$, $\alpha^i_{-1}(y) = x^i$. Now let us consider a new system of coordinates $\overline{x}^i = \overline{x}^i(x) = \alpha^i(x)$ in the domain Ω that is defined by the transformation of this domain. Let $A_i(x)$ be a covector field in Ω. In the new system of coordinates, one can consider the covector field $A_i(\overline{x})$. Let us see what this new covector field looks like in the initial system of coordinates. We have

$$\widetilde{A}_i(x) = A_j(\overline{x})\frac{\partial \overline{x}^j(x)}{\partial x^i} = A_j(\alpha(x))\frac{\partial \alpha^j(x)}{\partial x^i}.$$

Thus, we put any transformation of the domain Ω into correspondence with the transformation of the covector fields in accordance with the rule

$$\widetilde{A}_i(x) = A_j(\alpha(x))\frac{\partial \alpha^j(x)}{\partial x^i}.$$

For the scalar field and the symmetric covariant tensor field of the second rank, we have $\widetilde{\varphi}(x) = \varphi(\alpha(x))$,

$$\widetilde{g}_{ij}(x) = g_{kl}(\alpha(x))\frac{\partial \alpha^k(x)}{\partial x^i}\frac{\partial \alpha^l(x)}{\partial x^j}.$$

It is not difficult to write the laws of reparameterization for any fields. We mention important aspects of the R-symmetry. Let us consider some infinitesimal transformation of the domain Ω, setting $\alpha^i(x) = x^i + \xi^i(x)$ with condition that vector field $\xi^i(x)$ should be trivial on the boundary of Ω. It is not difficult to show that under the infinitesimal transformation difference $\widetilde{A}_i(x) - A_i(x) = \xi^k(x)\partial_k A_i(x) + A_k(x)\partial_i \xi^k(x)$ can be applied as variation in the stationary-action principle. It should be noted that, for example, the Maxwell equations in the external gravitational field can be written in any system of coordinates but they are not invariant with respect to the transformations of the R-symmetry, and only the Einstein–Maxwell equations are invariant with respect to the transformations of this symmetry. Thus, R-symmetry should be considered as a deep reason of special status of the gravitational field and the principle of universality of the gravitational interactions.

And last but not least, the derivatives of the gravitational potential g_{ij} must not enter into Lagrangian of field that is different from the gravitational one. A well-known example is the Lagrangian of the electromagnetic field. This demand is very important for the correct definition of the momentum of the field in question and is formulated as the concept of natural derivative. For example, the tensor of the electromagnetic field is a natural derivative of the vector potential. Other examples are given below.

From a geometrical point of view, the curvature tensor of the gravitational potential g_{ij} emerges naturally from the parallel displacement $\delta V^i = dV^i + \Gamma^i_{jk} dx^j V^k = 0$ and the commutator of the covariant derivatives $(\nabla_i \nabla_j - \nabla_j \nabla_i)V^k = R_{ijl}{}^k V^l$. The antisymmetrical tensor (trace) $R_{ijk}{}^k$ of the curvature tensor is trivial since $\partial_i \Gamma^k_{jk} - \partial_j \Gamma^k_{ik} = 0$. However, from $R_{ijl}{}^k$ we can create the symmetrical tensor $R_{jl} = R_{kjl}{}^k$ and the scalar $R = R_{jl} g^{jl}$ and define absolutely the l.h.s. of the Einstein equation $G_{ij} = T_{ij}$. You can see that Γ^i_{jk} is a natural derivative of g_{ij} and $R_{ijl}{}^k$ is a natural derivative of Γ^i_{jk}.

With this sequence of ideas in mind, it is natural to put forward an idea that a curvature tensor that defines the r.h.s. of the Einstein equation results from the general parallel displacement $\delta V^i = dV^i + P^i_{jk} dx^j V^k = 0$, where the connection P^i_{jk} is considered as a primary entity. To define a true status of the field P^i_{jk} and general relativity as well, let us define a general covariant gauge group G_c. To this end, we start from a natural general covariant generalization of the notion of the linear vector space putting that the vector fields are elements of L_g and multiplication on a real number is substituted by the multiplication on the scalar field. It is evident that $W^i(x) = a(x)U^i(x) + b(x)V^i(x)$ is again the element of L_g. The covector fields U_i are the elements of the dual space $\widetilde{L}_g$. The linear spaces L_g and $\widetilde{L}_g$ have notable properties. For a natural and constructive introduction of the linear operator, we do not need to introduce the n linear independent vector fields $E^i_\mu(x)$ as in the abstract theory of linear spaces, since the role of the linear operator is played here by the tensor field $S^i_j(x)$ of type (1,1). We consider the general covariant equations

$$\overline{V}^i(x) = S^i_j(x)V^j(x), \quad \overline{U}_i(x) = {S^{-1}}^j_i(x)U_j(x)$$

as the definition of the linear operator in the linear spaces in question. A product of two operators is defined in evident form $P^i_j(x) = S^i_k(x)T^k_j(x)$. Below it will be shown why we use for the covector fields operator ${S^{-1}}^k_j$ inverse to the operator S^i_j, $S^i_k{S^{-1}}^k_j = \delta^i_j$. We see that the linear spaces L_g and $\widetilde{L}_g$ are the spaces of the fundamental representation of the general covariant non-Abelian gauge group of internal symmetry. The parallel displacement of the vector fields $\overline{V}^i = S^i_j(x)V^j(x)$ and V^i can be produced only by a pair of connections $\overline{P}^i_{jk}$ and P^i_{jk}. From the law of parallel displacement we have

$$\overline{P}^i_{jk} = S^i_m P^m_{jn}{S^{-1}}^n_k + S^i_m\partial_j{S^{-1}}^m_k.$$

Hence, the general covariant gauge symmetry states that a physical configuration is not a given potential P^i_{jk} but rather a class of gauge equivalent potentials defined above. This symmetry essentially uniquely defines the dynamics of the general covariant gauge field and the r.h.s. of the Einstein equation and, hence, the nature of gravity. The general covariant gauge field in question will be called K-field.

We define gauge covariant derivatives as follows:

$$D_jV^i = \partial_jV^i + P^i_{jk}V^k, \quad D_jU_i = \partial_jU_i - P^k_{ji}U_k,$$

and have equations

$$\overline{D}_j\overline{V}^i = S^i_kD_jV^k, \quad \overline{D}_j\overline{U}_i = {S^{-1}}^k_iD_jU_k,$$

which establish the status of the vector and covector fields in the framework of the general covariant gauge group G_c. We define that the general covariant spinor field is the vector field V^i that is considered as the fundamental two-valued representation of the group G_c. And accordingly, the general covariant conjugated spinor field is the covector field that is considered as conjugated two-valued representation of the group G_c. It is evident that in this case S^i_j and $-S^i_j$ define different transformations of the group G_c. We consider a pair (U_i, V^j) as a general covariant bispinor field. The other vector and covector fields should be considered as invariants with respect to the transformations of the group G_c. We introduce two fundamental gauge invariant covector fields and one scalar field putting

$$\xi_i = U_l D_i V^l, \quad \eta_i = V^l D_i U_l, \quad \varphi = U_l V^l.$$

We have $\xi_i + \eta_i = \partial_i \varphi$. Now we establish general covariant and gauge invariant Lagrangian for our general covariant bispinor field (U_i, V^j) and derive equations for this field. Equations for the general covariant gauge field P^i_{jk} will be established in the next section. We put

$$\mathcal{L}_S = l_s^2 \left(\frac{1}{2}\xi_i \xi^i + \frac{1}{2}\eta_i \eta^i - \frac{1}{2}m^2 \varphi^2 \right),$$

where $\xi^i = g^{ij}\xi_j$, l_s and m are constants of the dimension of cm^{-1}, and hence (U_i, V^j) and action are dimensionless. Varying with respect to U_i and V^j, we derive the following system of equations:

$$\zeta^i D_i U_l + (\nabla_i \xi^i + m^2 \varphi) U_l = 0, \quad \zeta^i = \xi^i - \eta^i, \tag{1.1}$$

$$\zeta^i D_i V^j - (\nabla_i \eta^i + m^2 \varphi) V^j = 0, \tag{1.2}$$

where ∇_i is covariant derivative with respect to connection belonging g_{ij}. From this system of equations it is not difficult to derive two important consequences:

$$\nabla_i (\zeta^i \varphi) = 0, \tag{1.3}$$

$$D_i (\sqrt{g}\, \zeta^i U_l V^j) = 0, \quad g = -\mathrm{Det}(g_{ij}). \tag{1.4}$$

The commutator of the gauge covariant derivatives $(D_i D_j - D_j D_i) V^k = [D_i, D_j] V^k = H_{ijl}{}^k V^l$, where $H_{ijl}{}^k = \partial_i P^k_{jl} - \partial_j P^k_{il} + P^k_{in} P^n_{jl} - P^k_{jn} P^n_{il}$, gives the curvature tensor (the strength tensor) of the general covariant gauge field. It is evident that $H_{ijl}{}^k$ is the natural derivative of the potential P^i_{jk}. We also have that $\overline{H}_{ijl}{}^k = S^k_m H_{ijn}{}^m S^{-1}{}^n_l$. The antisymmetric tensor $F_{ij} = H_{ijk}{}^k = \partial_i P^k_{jk} - \partial_j P^k_{ik}$ (trace of the curvature tensor) is nontrivial here and should be considered separately from the irreducible (traceless) strength tensor $I_{ijl}{}^k = H_{ijl}{}^k - \frac{1}{4} H_{ijn}{}^n \delta^k_l$, $\quad I_{ijl}{}^l = 0$. It is easy to see that some indices have a double purpose. In view of this, in what follows we will use the natural matrix notation

$$\mathbf{S} = (S^k_l), \quad \mathbf{S}^{-1} = (S^{-1}{}^k_l), \quad \mathbf{P}_i = (P^k_{il}), \quad \mathbf{E} = (\delta^k_l),$$

$$\mathbf{H}_{ij} = (H_{ijl}{}^k), \quad \mathrm{Tr}\, \mathbf{H}_{ij} = H_{ijk}{}^k, \quad \mathbf{H}_{ij} = \partial_i \mathbf{P}_j - \partial_j \mathbf{P}_i + [\mathbf{P}_i, \mathbf{P}_j],$$

which allocates these indices. We also consider covector field U_i as matrix-row, vector field V^i as matrix-column, their product $U_l V^j$ as matrix and put

$$\mathbf{U} = (U_i), \quad \mathbf{V} = (V^i), \quad \mathbf{W} = (U_l V^j).$$

The transformations of the general covariant gauge field and the other fields take the simple form

$$\overline{\mathbf{P}}_i = \mathbf{S}\mathbf{P}_i \mathbf{S}^{-1} + \mathbf{S}\partial_i \mathbf{S}^{-1} = \mathbf{P}_i + \mathbf{S} D_i \mathbf{S}^{-1}, \quad \overline{\mathbf{H}}_{ij} = \mathbf{S}\mathbf{H}_{ij}\mathbf{S}^{-1},$$

$$\overline{D}_i\overline{\mathbf{H}}_{jk} = \mathbf{S}D_i\mathbf{H}_{jk}\mathbf{S}^{-1}, \quad \overline{\mathbf{W}} = \mathbf{SWS}^{-1},$$

where D_i is the gauge covariant derivative defined above and associated with general covariance and general covariant gauge symmetry; $D_i\mathbf{S} = \partial_i\mathbf{S} + \mathbf{P}_i\mathbf{S} - \mathbf{S}\mathbf{P}_i = \partial_i\mathbf{S} + [\mathbf{P}_i, \mathbf{S}]$ is the tensor field, $D_i\mathbf{H}_{jk} = \partial_i\mathbf{H}_{jk} + [\mathbf{P}_i, \mathbf{H}_{jk}]$ is not the tensor field but $D_i\mathbf{H}_{jk} + D_j\mathbf{H}_{ki} + D_k\mathbf{H}_{ij}$ is the tensor field, and the identity $D_i\mathbf{H}_{jk} + D_j\mathbf{H}_{ki} + D_k\mathbf{H}_{ij} = 0$ is general covariant.

To allocate the physical degrees of freedom of the K-field and disclose a mechanism of emergence of mass of this field, we introduce the important notion of the ground state. The ground state of the general covariant gauge field is defined as a solution of the equation $\mathbf{H}_{ij} = 0$. Let four linear independent vector fields E^i_μ be given. With this, one can construct purely algebraical components of the four covector fields E^μ_i, so that $E^i_\mu E^\mu_j = \delta^i_j$ holds valid. Setting $P^k_{il} = L^k_{il}$, where $L^k_{il} = E^k_\mu \partial_i E^\mu_l$ is a linear connection of the ground state, we get a general solution of the equation $\mathbf{H}_{ij} = 0$. For the ground state we have $\mathrm{Tr}\,(\mathbf{L}_i) = \partial_i \ln|p|$, where $p = \mathrm{Det}\,(E^\mu_i)$. Thus, we can define the ground state as any quadruplet of linear independent vector fields E^i_μ associated with the connection $L^k_{il} = E^k_\mu \partial_i E^\mu_l$. The ground state is invariant under the general covariant gauge transformations. Indeed, if the quadruplet of vector fields E^i_μ represents the ground state, then $\overline{E}^i_\mu = S^i_j E^j_\mu$ is the ground state as well, since $\overline{\mathbf{L}}_i = \mathbf{S}\mathbf{L}_i\mathbf{S}^{-1} + \mathbf{S}\partial_i\mathbf{S}^{-1}$.

The transition from the ground state to the excited one is characterized by the tensor of transition $T^i_{jk} = P^i_{jk} - L^i_{jk}$, with a simple (homogeneous) law of transformation $\overline{\mathbf{T}}_i = \mathbf{S}\mathbf{T}_i\mathbf{S}^{-1}$ and the irreducible tensor $\mathbf{Q}_j = \mathbf{T}_j - \frac{1}{4}(\mathrm{Tr}\,\mathbf{T}_j)\,\mathbf{E}$ with the trivial trace $\mathrm{Tr}\,\mathbf{Q}_j = 0$.

2. BASIC EQUATIONS

To derive equations of gravitodynamics, we establish the Lagrangian of K-field putting

$$\mathcal{L}_P = -\frac{1}{4}\,\mathrm{Tr}\,(\mathbf{I}_{ij}\mathbf{I}^{ij}) + \frac{\mu^2}{2}\,\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i), \quad \mathcal{L}_{em} = -\frac{1}{4}F_{ij}F^{ij},$$

where $\mathbf{I}^{ij} = g^{ik}g^{jl}\mathbf{I}_{kl}$, $\mathbf{Q}^i = g^{ik}\mathbf{Q}_k$, and μ is a constant of the dimension of cm^{-1}. Dimension of P^i_{jk} is equal to cm^{-1} and the action is dimensionless. The general covariant and gauge invariant Lagrangian $\mathcal{L}$ of all fields in question takes the form $\mathcal{L} = \mathcal{L}_P + \mathcal{L}_{em} + \mathcal{L}_s + \mathcal{L}_g$, where $\mathcal{L}_g = (l_g^2/2)R$ is the Lagrangian of the gravitational field and l_g is a constant of the dimension of cm^{-1}. By varying the Lagrangian $\mathcal{L}$ with respect to $\mathbf{P}_i$, the following equation holds:

$$\frac{1}{\sqrt{g}}D_i(\sqrt{g}\,\mathbf{I}^{ij}) + \mu^2\mathbf{Q}^j + \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,F^{ij})\,\mathbf{E} + \zeta^j\,\mathbf{W} = 0. \qquad (2.1)$$

Taking trace from equation (2.1), we find that

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,F^{ij}) + \frac{1}{4}\varphi\zeta^j = 0,$$

since $\mathrm{Tr}\,\mathbf{I}^{\mathrm{ij}} = \mathrm{Tr}\,\mathbf{Q}^{\mathrm{j}} = 0$. Hence,

$$\frac{1}{\sqrt{g}}D_i(\sqrt{g}\,\mathbf{I}^{ij}) + \mu^2\mathbf{Q}^j + \zeta^j\left(\mathbf{W} - \frac{1}{4}(\mathrm{Tr}\mathbf{W})\,\mathbf{E}\right) = 0. \tag{2.2}$$

From (2.2) it follows that $\mathbf{Q}^i$ has to satisfy the equation $D_i(\sqrt{g}\,\mathbf{Q}^i) = 0$, because $D_iD_j(\sqrt{g}\,\mathbf{I}^{ij}) = 0$, $D_i(\sqrt{g}\,\zeta^i\mathbf{W}) = 0$ and $\partial_i(\sqrt{g}\,\zeta^i\varphi) = 0$.

To make a clear and apparent conclusion from these results, we first of all mention that the strength tensor $\mathbf{I}_{ij}$ can be written in terms of the irreducible tensor $\mathbf{Q}_i$ only, since

$$\mathbf{I}_{ij} = \overset{o}{D}_i\,\mathbf{Q}_j - \overset{o}{D}_j\,\mathbf{Q}_i + \mathbf{Q}_i\mathbf{Q}_j - \mathbf{Q}_j\mathbf{Q}_i.$$

Here $\overset{o}{D}_i$ denotes the gauge covariant derivative with respect to the connection $\mathbf{L}_i$ of the ground state and, hence, $[\overset{o}{D}_i, \overset{o}{D}_j] = 0$. For the antisymmetric tensor F_{ij} we obtain

$$F_{ij} = \partial_i P^k_{jk} - \partial_j P^k_{ik} = \partial_i(L^k_{jk} + T^k_{jk}) - \partial_j(L^k_{ik} + T^k_{ik}) = \partial_i T^k_{jk} - \partial_j T^k_{ik},$$

since $\mathrm{Tr}\mathbf{L}_i = \partial_i \ln|p|$, $p = \mathrm{Det}(E^\mu_i)$. Thus, we can consider the tensor field Q^k_{il} with the constraints $Q^k_{ik} = 0$ and covariant vector field $A_i = T^k_{ik}$ as independent quantities, which obey the equations

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}\,F^{ij}) + \frac{1}{4}\varphi\zeta^j = 0, \quad F_{ij} = \partial_i A_j - \partial_j A_i,$$

$$\frac{1}{\sqrt{g}}\;\overset{o}{D}_i\;(\sqrt{g}\,\mathbf{I}^{ij}) + [\mathbf{Q}_i, \mathbf{I}^{ij}] + \mu^2\mathbf{Q}^j + \zeta^j\left(\mathbf{W} - \frac{1}{4}(\mathrm{Tr}\,\mathbf{W})\,\mathbf{E}\right) = 0.$$

We conclude that Einstein equation of the gravitodynamics takes the form

$$l_g^2 G_{ij} + T_{ij} = 0,$$

where $T_{ij} = P_{ij} + E_{ij} + S_{ij}$ and

$$P_{ij} = -\mathrm{Tr}\,(\mathbf{I}_{ik}\mathbf{I}_j{}^k) - g_{ij}\mathcal{L}_P + \mu^2\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}_j)$$

is the energy–momentum tensor of the K-field, $\mathbf{I}_j{}^k = \mathbf{I}_{jl}g^{kl}$,

$$E_{ij} = -F_{il}F_j{}^l - g_{ij}\mathcal{L}_{em}, \quad S_{ij} = l_s^2(\xi_i\xi_j + \eta_i\eta_j) - g_{ij}\mathcal{L}_s$$

are the energy–momentum tensor of the electromagnetic field and the energy–momentum tensor of the gauge covariant bispinor field (U_i, V^j), respectively. It is evident that the tensors P_{ij}, S_{ij}, E_{ij} are invariant under the transformations of the general covariant gauge group. The mass term $\mu^2\mathrm{Tr}\,(\mathbf{Q}_i\mathbf{Q}^i)$ is obtained by means which do not violate the general covariant gauge symmetry, and this is an important point for the renormalizability of the theory.

From our consideration it follows that the ground state E^i_μ represents nonphysical degrees of freedom and equation $S^i_j E^j_\mu = \delta^i_\mu$ is an evident and natural gauge fixing.

Now our goal is to establish the physical meaning of the constants l_g, l_s, μ, and m. To this end, let us remind that in the electrodynamics it is impossible to consider dimension of the vector potential equal to cm^{-1} without constant e that is called "electrical charge". In gravitodynamics the same situation takes place with respect to the general covariant gauge field P^i_{jk}, since $\mathbf{H}_{ij} = \partial_i \mathbf{P}_j - \partial_j \mathbf{P}_i + [\mathbf{P}_i, \mathbf{P}_j]$ and the so-called self-interaction is evident. We introduce the gravitational charges simply putting

$$e_g = m_g \sqrt{G}\,, \quad e_s = m_s \sqrt{G}\,.$$

Hence,

$$l_g^2 = \frac{c^4}{e_g^2 G}, \quad l_s^2 = \frac{c^4}{e_s^2 G}.$$

We also put

$$\mu = \frac{m_g c}{\hbar_g}, \quad m = \frac{m_s c}{\hbar_g},$$

where $\hbar_g$ is a gravitational Planck's constant and, hence,

$$\alpha_g = \frac{e_g^2}{\hbar_g c}, \quad \alpha_s = \frac{e_s^2}{\hbar_g c}$$

are coupling constants of the gravitodynamics. Since the ratio $e^2/e_g^2 = \alpha$, if $\hbar/\hbar_g = \alpha_g$, then we can estimate m_g as Planck's mass, but m_s stays arbitrary. Thus, three fields in question form one whole with the gravitationals field and represent the very fabric of reality.

CONCLUSIONS

Whilst the need for invisible matter was established almost a century ago, only its gravitational interaction has been confirmed so far. This is a very important observation, since it supports our statement that the gravitodynamicsis is the theory of this matter which is presented with the general covariant gauge field and the general covariant bispinor field. Thus, there is no reason for a plethora of models for this dark matter. From our consideration it is clear that the general covariant bispinor field can be considered as a source of the general covariant gauge field and its gauge invariant state, which we identify with the cosmic electromagnetic field. The last statement demands explanation. Of course, it is evident that the general covariant gauge field does not interact with fermionic matter, which forms important basis of our existence and our devices. But we can observe its gauge invariant state (cosmic electromagnetic field) with the help of our devices constructed from the fermionic matter. A theoretical reason for this is very simple. We can without any contradiction enter the electromagnetic field into the Lagrangian of the general covariant Dirac theory [4] as external field. Thus, from the observations we can conclude that the gauge invariant state of the general covariant gauge field represents the Cosmic Microwave Background. Since we now know the natural sources of the gravitational field at the large scale, the new status of the gravitational waves should be considered as well but this is a subject of a separate consideration. It is now clear that the investigation of the gravitodynamics as a closed gravitating system in the framework of quantum field theory is now an urgent problem because it can be renormalized.

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